Roll No. Total No. of Pages : 02

Total No. of Questions : 09

B.Tech.(CE)/(ECE)/(EE)/(Electrical & Electronics)/
B.Tech. (Electronics & Computer Engg.)/
B.Tech. (Electronics & Electrical)/(ETE) (2011 Onwards)
B.Tech.(Electrical Engg. & Industrial Control) (2012 Onwards)
B.Tech.(Electronics Engg.) (2012 Onwards)
(Sem.–3)

ENGINEERING MATHEMATICS – III

Subject Code : BTAM-301
Paper ID : [A1128]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

1. Write briefly :

a) State and prove Second Shifting property for Laplace Transform.

b) Find inverse Laplace Transform of $\dfrac{2(s+1)}{(s^2+2s+2)^2}$.

c) Find Laplace Transform of $t^2.\sinh t$.

d) State Rodrigue's formula for Legendre polynomials.

e) Using $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$, show that $J_{-1/2}=\sqrt{\dfrac{2}{\pi x}}\cos x$, where $J_n(x)$ is a Bessel function of First kind.

f) By eliminating the arbitrary function from $z = xy + f(x^2 - y^2)$, obtain a partial differential equation.

g) Find solution of homogeneous partial differential equation $2r - 5s + 2t = 0$.

h) If $f(z)$ is analytic and $Im\, f(z)$ is constant then show that $f(z)$ is constant.

i) Show that $\oint_C \dfrac{dz}{(z^2+4)^2}=\dfrac{\pi}{16}$, where $C : |z - i| = 2$.

j) Using the Residue theorem, evaluate $\oint_C \dfrac{e^z}{(z+1)^n}dz$, where $C : |z| = 2$.

SECTION-B

2. A periodic function of period 2 is defined as $f(x) = 1 + x, -1 < x < 1$. Obtain the Fourier series expansion of $f(x)$ and hence show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...$$

3. Using Laplace Transform, solve the given initial value problem

$$y'' + 6y' + 9y = 8te^{2t}, y(0) = 0, y'(0) = -1.$$

4. Find the power series solution about the point $x_0 = 2$ of the equation

$$y'' + (x-1)y' + y = 0.$$

5. Find the general solution of the partial differential equation $(x + y^2)p + yq = z + x^2$.

6. Solve the partial differential equation $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$, where $u(x, 0) = 4\ e^{-x}$ by the method of separation of variables.

SECTION-C

7. For Legendre polynomials $P_n(x)$, show that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0, & m \neq n. \\ \frac{2}{2n+1}, & m = n. \end{cases}$

8. a) For Bessel's function $J_n(x)$, show that $J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + ...) = 1.$

   b) Determine the angle of rotation at the point $z = (1 + i)/2$ under the mapping $w = z^2$. Find its scale factor also.

9. Find all possible Taylor's and Laurent series expansions of the function $f(z) = \frac{1}{(z+1)(z+2)^2}$ about the point $z = 1$.